PowerTeaching® Math
3rd Edition

Level 7 | Unit 4

Ratios and Proportions

Shift 1: Greater focus on fewer topics. In grade 7: Ratio and proportional relationships and arithmetic of rational numbers

unit guide

With Student and Assessment Pages

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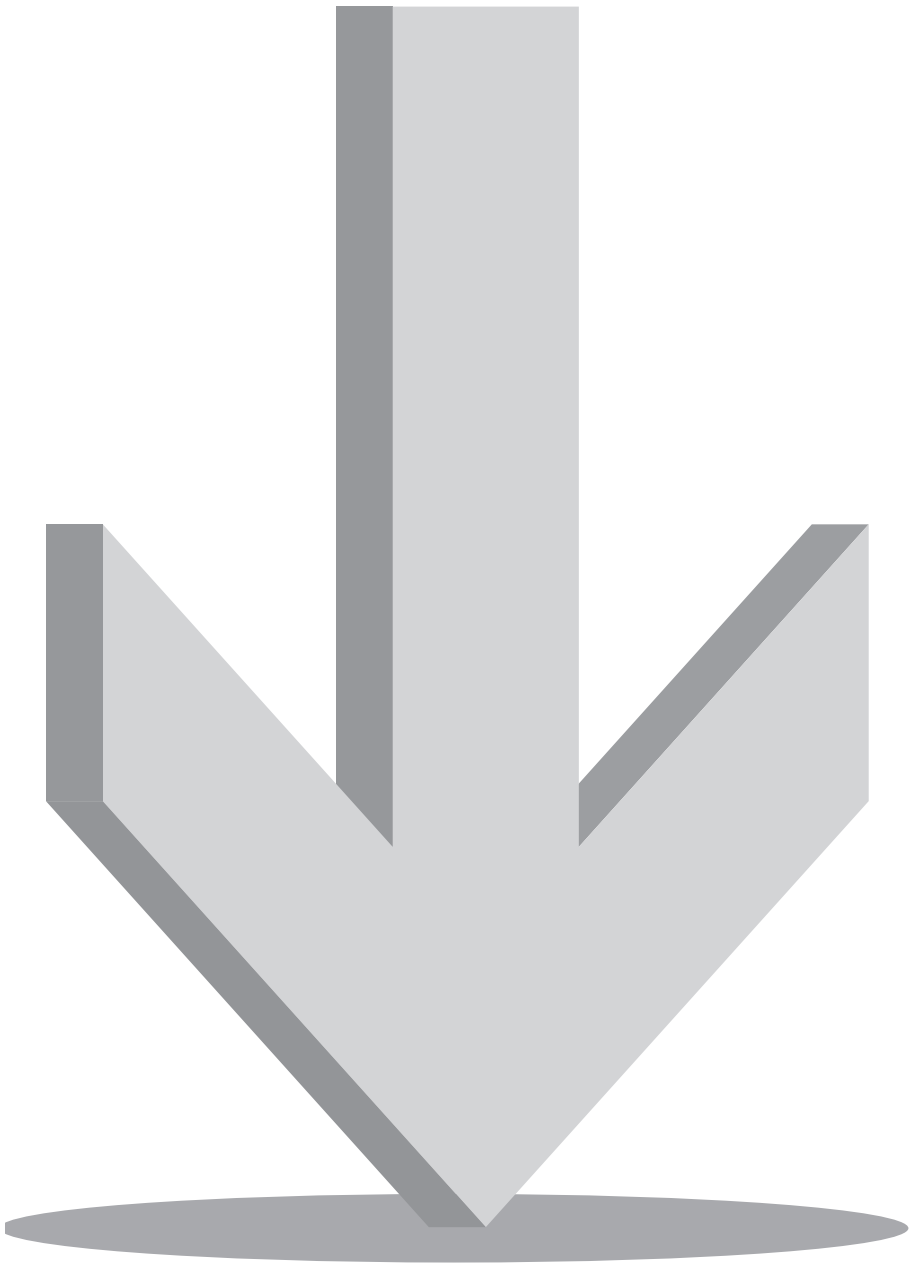

PowerTeaching Math 3rd Edition Unit Guide: Level 7

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Sample Table of Contents: Lists the cycles within each unit and the unit names which indicate the topics covered.

table of contents

This project was developed at the Success for All Foundation under the direction of Robert E. Slavin and Nancy A. Madden to utilize the power of cooperative learning, frequent assessment and feedback, and schoolwide collaboration proven in decades of research to increase student learning.

Shift 1: Greater focus on fewer topics. In grade 7: Ratio and proportional relationships and arithmetic of rational numbers

Level 7 | Unit 4: Ratios and Proportions

unit guide

Unit Overview

Unit 4 of grade 7 is about ratios and proportions and falls in the ratios and proportions strand. In grade 6, your students learned what ratios were and used fractions, tables, and tape diagrams to better understand ratios. In this unit, your students will apply their knowledge of unit rates by writing unit rates for fractional amounts and by using unit rates to solve problems. Your students will also use proportions to compare relationships. Then your students will learn how proportional relationships are one type of linear relationship.

Unit 4 has three cycles: Cycle 1—Unit Rates, Cycle 2—Proportional Relationships, and Cycle 3—Equations and Proportional Relationships.

Specific vocabulary for each unit helps prepare students to better explain concepts and formulate answers to problems.

Vocabulary introduced in this unit:

rate
unit rate
proportion
proportional relationship
constant of proportionality (*k*)
slope
independent variables
dependent variables
pattern

Think Like a Mathematician practices are derived from CCSS Mathematical Practice Standards.

Think Like a Mathematician practice(s) used in this unit:

Make sense of it.
Translate into math.
Defend and review.
Build a math model.
Use your math toolkit.
Be precise.
Find the patterns and structure.
Look for repetition.

Cycle 1—Unit Rates

Lesson 1: Basic Unit Rates
Find unit rates involving whole numbers and decimals. (CC 6.RP.A.2 and 7.RP.A.1; TEKS 7.b.4.B; VA SOL 7.4, 7.4CF)

Lesson 2: Unit Rates with Fractions
Find unit rates using data with fractions. (CC 7.RP.A.1; TEKS 7.b.4.B; VA SOL 7.4, 7.4CF)

Lesson 3: Problem Solving with Unit Rates
Solve multistep problems involving unit rates. (CC 7.RP.A.1; TEKS 7.b.4.B and D; VA SOL 7.4)

Each unit lists specific CCSS standards for every lesson.

Cycle 2—Proportional Relationships

Lesson 1: Defining Proportional Relationships
Identify proportional and nonproportional relationships. (CC 7.RP.A.2a; TEKS 6.b.4.A, 8.b.5.F; VA SOL 7.4, 7.6, 7.12)

Lesson 2: Solving Proportions
Use proportions to solve real-world problems. (CC 7.RP.A.2a; TEKS 7.b.4.A; VA SOL 7.4, 7.4CF)

Lesson 3: Proportions in Tables and Graphs
Identify proportionality in tables and graphs. (CC 7.RP.A.2a, d; TEKS 7.b.4.A; VA SOL 7.4, 7.4CF, 7.12)

Lesson 4: Problem Solving with Proportions 1
Solve multistep problems involving proportions. (CC 7.RP.A.2–3; TEKS 7.b.4.A and D; VA SOL 7.4)

Cycle 3—Equations and Proportional Relationships

Lesson 1: Constant of Proportionality
Identify the constant of proportionality. (CC 7.RP.A.2b; TEKS 7.b.4.C; VA SOL 7.4)

Lesson 2: Represent a Proportion as an Equation
Write equations for proportions, using *k* as the constant of proportionality. (CC 7.RP.A.2c; TEKS 7.b.4.A, C; VA SOL 7.4)

Lesson 3: Interpret Points of a Proportional Relationship

Explain what a point on a graph of a proportional relationship means in terms of a situation. (CC 7.RP.A.2d; TEKS 7.b.4.A and C; VA SOL 7.4, 7.12)

Lesson 4: Problem Solving with Proportions 2

Identify independent and dependent variables and solve real-world problems involving proportions. (CC 7.RP.A.2; TEKS 7.b.4.A and C; VA SOL 7.4, 7.12)

Lesson 5: Think Like a Mathematician: Find the Patterns and Structure 1

Find a pattern. (CC MP.7; TEKS 7.b.1.F)

Lesson 1: Basic Unit Rates

Vocabulary:
rate
unit rate

Materials:
none

Lesson Objective: Find unit rates involving whole numbers and decimals.

By the end of this lesson, students will:

- find unit rates and unit prices given whole number and decimal information;
- use words to demonstrate understanding of the rates and express the unit rates and unit prices in multiple ways; and
- compare unit rates and unit prices to find the greatest or least.

opening

(3 minutes) tp

get the goof sr

- Ask teams to begin Get the Goof.

TEACHER'S NOTE: Students may forget that when dividing with decimals, the divisor must be a whole number, so they need to move the decimal as many places as necessary to make this happen. The decimal then has to be moved an equal number of places in the dividend.

When Audrey solved 5.672 ÷ 1.3, she got the quotient 0.4363. What's wrong with her thinking?

Random Reporter Rubric | Possible Answer

Answer: Audrey's quotient is incorrect because the decimal is in the wrong place. The decimal point should be one place to the right for a quotient of 4.363.

Explanation: Because Audrey was dividing two decimals, I knew that she had to move the decimals one place to the right, so the divisor becomes 13 and the dividend 56.72.

Math Practice: I know that when I divide by a decimal, the decimal point has to move to the right until the divisor is a whole number (TLM #3). Then the decimal point in the dividend has to move to the right the same number of times as in the divisor.

Standard for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

Speaking and Listening

SL.7.3: Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence.

Each time Random Reporter is used, students are asked not only to explain their thinking, but also to state how their responses relates to the Think Like a Mathematician practices they used when solving the problem.

active instruction

(10–15 minutes) tp

set the stage

- Distribute team score sheets. Have students review their scores and set new team goals in lesson 1.
- Post and present the lesson objective: Today you will find unit rates and unit prices with decimals and whole numbers.
- Ask students to write this cycle's vocabulary words in their notebooks: *rate, unit rate*.
- Remind students how to earn team celebration points.

interactive instruction and guided practice

Shift 2: Coherence and linking topics and thinking across grades

tps

- Use **Think-Pair-Share** to have students answer the following question: **You've probably heard of the terms *rate* and *unit rate*. What is a rate? What is a unit rate? Can you give an example of each?**
- Randomly select a few students to share. **Possible answer:** A rate is a ratio that compares two different units of measure. A unit rate compares a quantity to 1 unit of another measure. An example of a rate is paying $1 for every 16 oz of pasta. An example of a unit rate is miles per hour.

ta

- Use a **Think Aloud** to model finding the rate and unit rate.

Find the rate and unit rate.

4 layers

Stella's family drove 184.3 miles to the beach for the family vacation. The trip took 4.8 hours. Eli's family drove 212.5 miles to the mountains for their vacation. Their trip took 5.1 hours. Which family reached their destination at a faster rate?

Ratios and Proportional Relationships
7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Remember that in the last unit, we discussed multiplying and dividing rational numbers. This will help us in this unit as we discuss ratios and proportional relationships. A ratio is a different way of showing division, so keep that in mind as we discuss rate and unit rate.

Let me look at this problem. It wants me to find the rate for each family so I can figure out who reached their destination faster. A good mathematician makes sure he or she can define and identify any terms and units he or she will use before beginning a problem. I notice that in the problem, I have some numbers with different units of measure—miles and hours. This will be important.
Show layer 1.

Stella's family:

$\frac{184.3 \text{ mi}}{4.8 \text{ hr}}$, 184.3 miles in 4.8 hours

Eli's family:

$\frac{212.5 \text{ mi}}{5.1 \text{ hr}}$, 212.5 miles in 5.1 hours

Since a rate is a type of ratio, I can write the information for each family as a fraction. Stella's family drove 184.3 miles in 4.8 hours, which I can write as $\frac{184.3 \text{ mi}}{4.8 \text{ hr}}$. Eli's family drove 212.5 miles in 5.1 hours, which I can write as $\frac{212.5 \text{ mi}}{5.1 \text{ hr}}$. Show layer 2.

But who drove at a faster rate? It's hard to compare their driving because I have different numerators and denominators. Show layer 3.

Stella's family:

$\frac{184.3 \text{ mi}}{4.8 \text{ hr}}$, 184.3 miles in 4.8 hours

$\frac{38.4 \text{ mi}}{1 \text{ hr}}$, 38.4 miles per hour (mph)

Eli's family:

$\frac{212.5 \text{ mi}}{5.1 \text{ hr}}$, 212.5 miles in 5.1 hours

$\frac{41.7 \text{ mi}}{1 \text{ hr}}$, 41.7 miles per hour (mph)

I can solve this by finding the unit rate, which will give me the miles that Stella and Eli drove over 1 hour. I just divide the number of miles by the number of hours it took to drive them. So 184.3 ÷ 4.8 is 38.4 miles per hour (mph), and 212.5 ÷ 5.1 is 41.7 mph. Show layer 4.

Now that Stella's and Eli's rates match as a distance traveled per one hour, I can easily compare them. Eli's family drove at a faster rate than Stella's. His family drove 41.7 miles in one hour, which is faster than driving 38.4 miles in one hour.

Speaking and Listening

SL.7.1.C: Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed.

tps

- Use **Think-Pair-Share** to have students answer the following question: **Why do we need to find the unit rate? How can we use the unit rate to find other information about the situation?**
- Randomly select a few students to share. **Possible answer:** Finding the unit rate gives us information we can understand. For example, I know how fast Stella's and Eli's cars were driving on the road now. A car shows you the speed at which you are driving so you know how fast you are going. I can raise or lower the unit rate to calculate how many fewer or more hours Stella's and Eli's trips could have taken.

Standard for Mathematical Practice

1: Make sense of problems and persevere in solving them.

th

- Use **Team Huddle** to have teams practice finding the rate and unit rate.

Speaking and Listening

SL.7.1: Engage effectively in a range of collaborative discussions (one-on-one, in groups and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly.

1) Shoppers can buy a five-ounce tube of brand A toothpaste for $5.29 or a three-ounce tube of brand B for $4.37. Write a unit price that describes the cost per ounce for each type of toothpaste. Which is the better deal? Explain your thinking.

Random Reporter Rubric | Possible Answer

Answer: Brand A: $\frac{\$1.06}{1 \text{ oz}}$ Brand B: $\frac{\$1.46}{1 \text{ oz}}$

Brand A is the better deal because it costs the least per ounce.

Explanation: I divided the price of each tube of toothpaste by the ounces of toothpaste in each tube to find the unit rate.

Math Practice: I paid attention to precision (TLM #6) because I had to compare the prices of two different items with different sizes. I knew that my unit rate had to be accurate to determine the better deal.

Standard for Mathematical Practice

6: Attend to precision.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

team mastery

(10–15 minutes) tp sr

Shift 3: Rigor: Pursue conceptual understanding, procedural fluency, and application with equal intensity.

- Ask students to follow the Team Mastery student routine.
- Circulate and use the following questions to prompt discussions.
  - Describe this rate in words.
  - How do you know that this is a unit rate?
  - How did you find this rate/unit rate?
  - How does knowing the unit rate help you understand the situation?
- When there are 5 minutes left in Team Mastery, prompt teams to prepare for the Lightning Round. Have teams discuss one Team Mastery problem that the whole team has completed.
- Award team celebration points for good team discussions that demonstrate 100-point responses.

Speaking and Listening
SL.7.1.B: Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed.

lightning round

(10 minutes) tp sr

- Tell students the Team Mastery problem that you will use for the Lightning Round.

4) Oscar claims that he is a faster painter than Paloma. Oscar painted a 111.6-square-foot wall in 2.2 hours, and Paloma painted a 127.33-square-foot wall in 2.4 hours. Write unit rates that describe how fast Oscar and Paloma paint per square foot. Is Oscar right?

Random Reporter Rubric | Possible Answer

Answer: Oscar: $\frac{50.73 \text{ ft}^2}{1 \text{ hr}}$ Paloma: $\frac{53.05 \text{ ft}^2}{1 \text{ hr}}$

Oscar is not right; Paloma is a faster painter than Oscar.

Explanation: I divided the area of wall that each person painted by the time it took him or her to paint the wall to find the unit rate. Then I could compare their speeds.

Math Practice: I paid attention to precision (TLM #6) because I had to compare the speeds of the painters of two different walls with different times. I knew that my unit rate had to be accurate to describe how fast Oscar and Paloma paint per square foot.

Standard for Mathematical Practice
6: Attend to precision. Students use rubrics to assess the completeness and clarity of their oral and written explanations. They will also critique the explanations of their peers.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Tell students that it's time to power up Random Reporter. Use the layers on the page to guide discussion.
- Award team celebration points.

Speaking and Listening
SL.7.4: Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.

celebration

(2 minutes) tp sr

- Record team celebration points on the poster.
- Have the top team choose a cheer.
- Assign homework, and remind students about the Vocabulary Vault.
- Ask students to follow the Quick Check student routine. (optional)

Jessica spent $48.79 for 3 CDs and $70.52 for 5 DVDs. Write a unit price to describe the cost of each type of item. Which cost more per item, the CDs or DVDs?

Possible answer: CD: $\frac{\$16.26}{1\text{ CD}}$ DVD: $\frac{\$14.10}{1\text{ DVD}}$

The CDs cost more per item.

Lesson 2: Unit Rates with Fractions

Vocabulary:
none

Materials:
none

Lesson Objective: Find unit rates using data with fractions.

By the end of this lesson, students will:

- find unit rates with fractions, which may include one or two fractions;
- use words to demonstrate understanding of the rates and express the unit rates and unit prices in multiple ways; and
- compare unit rates and unit prices.

opening

(3 minutes) tp

get the goof sr

th

- Ask teams to begin Get the Goof.

> Omar walked a distance of 6.75 yards in 5.3 seconds. He said that he walked 0.78 yards per second. What is wrong with his thinking?
>
> **Random Reporter Rubric** | Possible Answer
>
> **Answer:** Omar found the unit rate incorrectly. Omar walked 1.3 yards per second.
>
> **Explanation:** He should have divided the total yards by the total time in seconds.
>
> **Math Practice:** I used TLM #3 to figure out what was wrong with Omar's thinking. Since Omar was looking for a unit rate, I knew that he needed to find how many yards he walked in 1 second. A unit rate compares a quantity (the yards Omar walked) to 1 unit of measure (the seconds it took to walk).

Standard for Mathematical Practice
3: Construct viable arguments and critique the reasoning of others.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

homework check sr

- Ask teams to do a homework check.
- Confirm the number of students who completed the homework on each team.
- Poll students on their team's understanding of the homework.
- Award team celebration points.
- Collect and grade homework once per cycle. Record individual scores on the teacher cycle record form.

Speaking and Listening
SL.7.1.A: Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion.

active instruction

(10–15 minutes) tp

set the stage

- Post and present the lesson objective: At the end of this lesson, you will be able to find the unit rate using data with fractions.
- Remind students how to earn team celebration points.

interactive instruction and guided practice

- Show the Q&A video: "Find Unit Rate with Fractions." In the video, Quincy and Andie work to figure out Andie's current running pace and the pace she wants to run for an upcoming 10k race.
- Use a **Think Aloud** to model calculating unit rates with fractions.

Calculate unit rates with fractions.

3 layers

Ray walks $\frac{9}{10}$ mile in $\frac{1}{3}$ hour on the treadmill. At what unit rate is Ray walking?

Shift 3: Rigor: Pursue conceptual understanding, procedural fluency, and application with equal intensity.

In the last lesson, we looked at rate and unit rate with decimals. Today we will look at them using fractions. This can be confusing because we write rates as fractions since they are a type of ratio. Stacking fractions to calculate a unit rate may look confusing, but we can use what we've learned about multiplying and dividing rational numbers to help us make sense of these problems.

Let me look at this problem. I need to find the rate at which Ray walks on the treadmill. Show layer 1.

I can express this as the fraction $\frac{\frac{9}{10}\text{ mi}}{\frac{1}{3}\text{ hr}}$. Show layer 2.

Ray walks $\frac{9}{10}$ of a mile every $\frac{1}{3}$ hour. That's hard to think about and doesn't tell me how fast he is going. I can convert this into a unit rate to make it easier to understand. Show layer 3.

$$\frac{\frac{9}{10}\text{ mi}}{\frac{1}{3}\text{ hr}} \cdot \frac{\frac{3}{1}}{\frac{3}{1}} = \frac{\frac{27}{10}}{1} = \frac{2.7\text{ mi}}{1\text{ hr}} \text{ or 2.7 miles per hour}$$

Since I need to divide $\frac{9}{10}$ by $\frac{1}{3}$ to find the unit rate, I can use what I know about dividing fractions to solve this. I can write it as $\frac{9}{10}\text{ mi} \cdot \frac{3}{1\text{ hr}}$.

It's important to remember that because I am looking for the miles per hour that Ray walked, I need to keep my hour unit in the denominator, even though I am multiplying by the inverse. So my product is $\frac{27\text{ mi}}{10\text{ hr}}$ which equals $\frac{2.7\text{ mi}}{1\text{ hr}}$, or 2.7 mph.

- Use **Think-Pair-Share** to have students answer the following question: **Why do you think it might be useful to find the unit rate or unit price when looking at fractions? For example, if we know that $\frac{2}{5}$ ounces of silver sells for $11.23, then how might the unit price be more helpful?**
- Randomly select a few students to share. **Possible answer:** The unit price gives us the price for a round measurement. The unit price of $11.23 for $\frac{2}{5}$ comes out to being $28.08 per 1 ounce of silver. We can better estimate prices when we know the unit rate.
- Use **Team Huddle** to have teams practice calculating unit rates with fractions.

1) Alex rode his bike $10\frac{3}{4}$ miles in $\frac{7}{10}$ hour. Kenya rode her bike $6\frac{1}{2}$ miles in $\frac{1}{2}$ hour. Write unit rates that describe their speeds. Who rode faster?

Random Reporter Rubric | Possible Answer

Answer: Alex = 15.36 mph; Kenya = 13 mph; Alex rode faster.

Explanation: Alex: $\frac{10\frac{3}{4}\text{ mi}}{\frac{7}{10}\text{ hr}} \cdot \frac{\frac{10}{7}\text{ hr}}{\frac{10}{7}\text{ hr}}$; Kenya: $\frac{6\frac{1}{2}\text{ mi}}{\frac{1}{2}\text{ hr}} \cdot \frac{\frac{2}{1}\text{ hr}}{\frac{2}{1}\text{ hr}}$; I calculated the unit rates for Alex and Kenya by dividing their total miles by their total times.

Math Practice: I modeled this problem (TLM #4) by writing a number sentence for both Alex's and Kenya's unit rates. I knew that to find the unit rates, I needed to write the ratios as a quantity over 1 unit of time.

Standard for Mathematical Practice
4: Model with mathematics.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

team mastery

(10–15 minutes) tp sr

- Ask students to follow the Team Mastery student routine.
- Circulate and use the following questions to prompt discussions.
  - What values will help you find this unit rate/unit price?
  - Describe the relationship between the values in this situation.
  - How do you find the unit rate when there is a fraction in the denominator?
  - What is the rate?
  - What is the unit rate?
- When there are 5 minutes left in Team Mastery, prompt teams to prepare for the Lightning Round. Have teams discuss one Team Mastery problem that the whole team has completed.
- Award team celebration points for good team discussions that demonstrate 100-point responses.

lightning round

(10 minutes) tp sr

- Tell students the Team Mastery problem that you will use for the Lightning Round.

3) Mrs. Wu finds that $\frac{3}{5}$ bag of mulch will cover $\frac{2}{3}$ square yard of garden. Write a unit rate that describes the number of bags of mulch needed to cover 1 square yard of garden.

Random Reporter Rubric | Possible Answer

Answer: $\frac{\frac{9}{10}\text{ bag}}{1\text{ yd}^2}$; it will take $\frac{9}{10}$ a bag of mulch for every 1 square yard of Mrs. Wu's garden.

Explanation: $\frac{\frac{3}{5}\text{ bag}}{\frac{2}{3}\text{ yd}^2} \cdot \frac{\frac{3}{2}\text{ yd}^2}{\frac{3}{2}\text{ yd}^2}$; I found the unit rate—the number of bags per square yard—by dividing the number of bags by the area.

Math Practice: I paid attention to precision (TLM #6) because I know that the unit rate is a ratio of a quantity of bags over 1 unit of yards. So I created a number sentence to help me solve the problem.

Standard for Mathematical Practice
6: Attend to precision.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

celebration

(2 minutes) tp

- Record team celebration points on the poster.
- Have the top team choose a cheer.
- Assign homework, and remind students about the Vocabulary Vault.
- Ask students to follow the Quick Check student routine. (optional)

Mrs. Robinson bought $2\frac{3}{8}$ pounds of caramel pecans for \$32.79 and 4.55 pounds of vanilla fudge for \$50.00. Which candy was more expensive per pound?

Possible answer: The caramel pecans were more expensive per pound. The caramel pecans were \$13.81 per pound, and the vanilla fudge was \$10.99 per pound.

Lesson 3: Problem Solving with Unit Rates

Vocabulary:
none

Materials:
none

Lesson Objective: Solve multistep problems involving unit rates.

By the end of this lesson, students will:

- solve complex word problems involving unit rates; and
- use rates to make predictions about future events.

opening

(3 minutes) tp

get the goof sr

th

- Ask teams to begin Get the Goof.

> Asia wants to divide two-thirds of a cake among 4 people. How much cake does each person get?
>
> Asia figures each person gets $2\frac{2}{3}$ of the cake. What is wrong with her thinking?
>
> **Random Reporter Rubric** | Possible Answer
>
> **Answer:** Asia did not find the unit rate correctly. The correct answer is $\frac{1}{6}$.
>
> **Explanation:** To find the unit rate, I had to find the amount of cake for 1 person, so I divided $\frac{2}{3}$ by 4. To do that, I multiplied $\frac{2}{3}$ by $\frac{1}{4}$ (the reciprocal of 4).
>
> **Math Practice:** I know that the amount of cake that each person should get should be less than $\frac{2}{3}$. My answer makes sense because I divided $\frac{2}{3}$ into 4 equal parts (TLM #3).

Standard for Mathematical Practice
3: Construct viable arguments and critique the reasoning of others.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

homework check sr

- Ask teams to do a homework check.
- Confirm the number of students who completed the homework on each team.
- Poll students on their team's understanding of the homework.
- Award team celebration points.
- Collect and grade homework once per cycle. Record individual scores on the teacher cycle record form.

active instruction

(10–15 minutes) tp

set the stage

- Post and present the lesson objective: Today you will problem solve with rates and unit rates.
- Remind students how to earn team celebration points.

interactive instruction and guided practice

ta

- Use a **Think Aloud** to model finding rates and unit rates in real-world problems.

Find rates and unit rates in real-world problems.

5 layers

Anthony needs to buy some notebooks and wants to get the best buy.

Brand	Number of pages per notebook	Number of notebooks	Cost in dollars for 5 notebooks
A	150	5	7.50
B	100	5	4.70
C	150	5	8.75
D	240	5	9.20

Standard for Mathematical Practice
4: Model with mathematics.

We use rates and unit rates to solve problems all the time. For example, we might use unit rates to tell whether something on sale at the store is really a good deal. Let's look at a real-world situation where we can problem solve with rates and unit rates. Show layer 1.

What is happening in this problem? I see that Anthony wants to know the best deal between four brands of notebooks. Finding the unit rate for each will tell me whether the cheapest deal on five notebooks is really the best deal. Show layer 2.

What do I know, and how can this help me? I know the price of 5 notebooks for each brand and the number of pages in each notebook. I can take this information from the chart and translate it into math to help me solve the problem. Show layer 3.

Brand A: $\frac{\$7.50}{5 \text{ notebooks}} = \frac{\$1.50}{1 \text{ notebook}}$

Brand B: $\frac{\$4.70}{5 \text{ notebooks}} = \frac{\$0.94}{1 \text{ notebook}}$

Brand C: $\frac{\$8.75}{5 \text{ notebooks}} = \frac{\$1.75}{1 \text{ notebook}}$

Brand D: $\frac{\$9.20}{5 \text{ notebooks}} = \frac{\$1.84}{1 \text{ notebook}}$

First, I need to find the cost of each notebook. So I'll divide the total cost of 5 notebooks by 5 to get my unit rate for each brand of notebook. Show layer 4.

Brand A: $\frac{\$7.50}{5 \text{ notebooks}} = \frac{\$1.50}{1 \text{ notebook}} \cdot \frac{1 \text{ notebook}}{150 \text{ pages}} = \frac{\$0.01}{1 \text{ page}}$

Brand B: $\frac{\$4.70}{5 \text{ notebooks}} = \frac{\$0.94}{1 \text{ notebook}} \cdot \frac{1 \text{ notebook}}{100 \text{ pages}} = \frac{\$0.0094}{1 \text{ page}}$

Brand C: $\frac{\$8.75}{5 \text{ notebooks}} = \frac{\$1.75}{1 \text{ notebook}} \cdot \frac{1 \text{ notebook}}{150 \text{ pages}} = \frac{\$0.011}{1 \text{ page}}$

Brand D: $\frac{\$9.20}{5 \text{ notebooks}} = \frac{\$1.84}{1 \text{ notebook}} \cdot \frac{1 \text{ notebook}}{240 \text{ pages}} = \frac{\$0.008}{1 \text{ page}}$

If I stopped at cost per notebook, I would tell Anthony that brand B was his best buy at $0.94. But since I know how many pages are in each notebook, I can find the cost per page. So I can divide the cost of each notebook by the number of pages in it. Show layer 5.

So even though brand B seemed like a good buy based on the cost per notebook, I see that brand D is the winner since it only costs $0.008 per page. This is the best deal he can get. Finding the unit rates helped me see how something that appears to be the best deal at first might not be the cheapest.

tps

- Use **Think-Pair-Share** to have students answer the following question: **What other situation might be described by two unit rates such as the cost per 1 notebook and the cost per 1 page?**
- Randomly select a few students to share. **Possible answer:** A situation such as buying cases of water bottles could be described by two unit rates. You could have the number of bottles in the package and the number of ounces per bottle.

th

- Use **Team Huddle** to have teams practice problem solving with unit rates.

TEACHER'S NOTE: A rubric answer is only provided for 1a of the question.

1) To qualify for the state swimming competition in the 100-meter freestyle race, swimmers must have an average speed of 1.7 meters per second in the trial races. The results of the three trials for each swimmer are shown below.

D. Hernandez	Finishing time (sec)
Trial 1	$57\frac{3}{10}$
Trial 2	$61\frac{1}{5}$
Trial 3	$58\frac{4}{5}$

L. Kim	Finishing time (sec)
Trial 1	$58\frac{7}{10}$
Trial 2	$58\frac{1}{2}$
Trial 3	$58\frac{1}{5}$

M. Johnson	Finishing time (sec)
Trial 1	$53\frac{1}{2}$
Trial 2	$59\frac{1}{5}$
Trial 3	$60\frac{1}{10}$

a. What is the average speed for each swimmer?

Random Reporter Rubric | Possible Answer for 1a

Answer: D. Hernandez: 1.69 m/s; L. Kim: 1.71 m/s; M. Johnson: 1.74 m/s.

Explanation: I added the times for each swimmer together and divided by 3 to find the average race time. Then I divided the total length of the race, 100 meters, by each swimmer's average time to find his or her average speed.

Math Practice: I translated the information from the charts into math (TLM #2) to solve the problem. I knew that I had to find the average finish times before I could find the unit rate, or average speed, of each swimmer.

Standard for Mathematical Practice
2: Reason abstractly and quantitatively.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

team mastery

(10–15 minutes) tp sr

- Ask students to follow the Team Mastery student routine.
- Circulate and use the following questions to prompt discussions.
  - How will finding the rate and/or the unit rate help you answer this question?
  - What is the rate?
  - What is the unit rate/price?
  - How do the different rates in this problem compare?
  - Why do you think the rates in this problem are different?
- When there are 5 minutes left in Team Mastery, prompt teams to prepare for the Lightning Round. Have teams discuss one Team Mastery problem that the whole team has completed.
- Award team celebration points for good team discussions that demonstrate 100-point responses.

lightning round

(10 minutes) tp sr

- Tell students the Team Mastery problem that you will use for the Lightning Round.

3) Caitlin keeps a record of when she babysits and how much she earns. Use Caitlin's babysitting data to answer the following questions.

> January 5th: worked 2 hours and earned $17.00
>
> January 12th: worked 5 hours and earned $42.50
>
> February 2nd: earned $34.00
>
> Upcoming job: February 19th: 3.5 hours

b. What unit rate describes what Caitlin charges for each hour of babysitting? Explain your thinking.

Random Reporter Rubric | Possible Answer for 3b

Answer: Caitlin earns $8.50 per hour.

Explanation: I used the information from January 5th to find the unit rate. I divided the total number of hours that Caitlin worked that day by what she was paid for her work.

Math Practice: I translated the information from Caitlin's records into math (TLM #2) to figure out how much she is paid per hour. I also checked my answer against my graph, and it made sense.

Standard for Mathematical Practice
2: Reason abstractly and quantitatively.

- Use **Random Reporter** to have teams share. Use the Random Reporter rubric to evaluate responses and give feedback.
- Record individual scores on the teacher cycle record form.
- Award team celebration points.

celebration

(2 minutes) tp

- Record team celebration points on the poster.
- Have the top team choose a cheer.
- Assign homework, and remind students about the Vocabulary Vault.
- Ask students to follow the Quick Check student routine. (optional)

A typical car engine contains about 5 quarts of motor oil.

a. If a car leaks $\frac{2}{32}$ quart of oil in $\frac{5}{24}$ of a day, what is the rate of oil loss per day?

b. If the car's owner adds $\frac{1}{2}$ quart of oil to the engine at the start of each week, how much oil will the car have at the end of the second week?

c. Will the engine run out of oil? If so, when? Explain your thinking.

Possible answers:

a. $\frac{3}{10}$ quarts per day.

b. There will be $1\frac{4}{5}$ quarts of oil remaining at the end of the second week.

c. Yes, the car's engine will run out of oil in 3 weeks and 3 days.
Possible explanation: I subtracted the total amount lost each day to determine whether the engine will run out of oil. Then I added the amount of oil added at the start of each week.

Assessment Day: Cycle Check on Unit Rates

Materials:
extra blank copies of the assessment

Lesson Objective: Demonstrate mastery of cycle content.

assessment

(20–30 minutes) tp

- Confirm the number of students who completed the homework on each team. Award team celebration points.
- Remind students that the test is independent work.
- Distribute the tests so students can preview the questions.
- Tell students the number of minutes they have for the test and that they may begin. Give students a 5-minute warning before the end of the test.
- Collect the tests.

team reflection

(5 minutes) tp sr

- Display or hand out blank copies of the test.
- Explain or review, if necessary, the student routine for team discussions after the test.
- Award team celebration points.

prep points

(5–10 minutes) tp

- Assign prep points for each team for the five questions indicated (1, 3, 5, 7a, 8b).
- Score individual tests when convenient.

vocabulary vault

(2 minutes) tp

- Randomly select vocabulary vouchers, and award team celebration points.
- Ask students to record the words that they explain on their team score sheets.

team scoring

(5 minutes) tp sr

- Guide the class to complete the team scoring on their team score sheets.

celebration

(2 minutes) tp

- Announce team statuses, and celebrate.
- Poll teams about how many times they have been super teams. Celebrate those teams, and encourage all teams to work toward super team status during the next cycle.
- Show the "Everyone Participates: Part 1" team cooperation video, which includes interviews with students and their classroom interactions as they practice having everyone in the team participate.
- Use **Think-Pair-Share** to have students discuss how this team cooperation goal can help them reach super team status. Randomly select students to share responses.

cycle check

1) This is Shaundra's receipt from her trip to the grocery store. What unit price did she pay for 1 pound of apples?

XYZ Grocery
Jul 22
"Always the best prices!"

Milk gal.	2.79
Bread	1.99
Apples @ 6 lbs	11.94
Total	

Ratios and Proportional Relationships
7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

2) Jake and his brother Franco both have the flu. Jake's temperature rose 3.4°F over 4 hours. Franco's temperature rose 5.2°F over 6 hours. Write a unit rate to describe the rise in temperature for each brother. Whose temperature had the greatest rate of increase?

3) A snail crawled 0.015 mile in $\frac{1}{2}$ hour, and a sloth moved 0.375 mile in $\frac{3}{8}$ hour. Write a unit rate to describe each animal's speed. Which is the slower animal?

4) Keisha buys $\frac{2}{3}$ dozen bagels for $4.80. Write a unit price to describe the cost of 1 dozen bagels.

5) The water flow from a local spring can fill $\frac{1}{6}$ gallon in 5 minutes. Write a unit rate to describe how quickly water flows from the spring each minute.

6) Henry is making dog cookies for his dogs. Write a unit rate to describe how much flour Henry would use to bake 1 dog cookie.

Best Dog Cookies Ever
makes 5 dog cookies

$1\frac{1}{2}$ cups (c) oats
$2\frac{3}{4}$ c flour
2 tbsp dried mint
$\frac{2}{3}$ c dried milk
$\frac{1}{2}$ tsp salt
2 eggs
$1\frac{1}{2}$ c peanut butter

Shift 3: Rigor: Pursue conceptual understanding, procedural fluency, and application with equal intensity.

7) A painter paints a family room in $2\frac{1}{4}$ hours. The family room is 10.5 feet by 12.3 feet.

   a. Write a unit rate that describes the time it took the painter to paint 1 square foot.

   b. If he continues at that rate, how long would it take the painter to paint a bedroom that is 114.8 square feet?

   c. The painter charged $56.25 for the total hours it took to paint the family room. At that rate, how much would he charge to paint the bedroom?

8) Alisa's class of 28 students and 1 teacher is taking a field trip to the natural history museum in the state capital. The distance from Alisa's town to the state capital is 225 miles.

Transportation	Travel Time (hours)	Cost
Train	$3\frac{2}{5}$	Groups of 21–30 people cost $970. Groups of 31–40 people cost $1,100.
Bus	4.09	Each person costs $33.50.

   a. Write a unit rate to describe the average speed at which the train and bus travel on their way to the state capital.

   b. Write a unit price to describe the cost for Alisa's class to travel on a train and a unit price to describe the cost for her class to travel by bus.

   c. Should Alisa's class travel by train or bus? Explain your thinking.

cycle check answers

Lesson 1: Find unit rates involving whole numbers and decimals. [30 points]

1) $\frac{\$1.99}{1 \text{ lb}}$; Shaundra paid $1.99 for each pound of apples. [15 points]

2) Jake's temperature: $\frac{0.85°F}{1 \text{ hr}}$ Franco's temperature: $\frac{0.87°F}{1 \text{ hr}}$

Franco's temperature had the greatest rate of increase. [15 points]

Lesson 2: Find unit rates using data with fractions. [34 points]

3) Snail: $\frac{0.03 \text{ mi}}{1 \text{ hr}}$ Sloth: $\frac{1 \text{ mi}}{1 \text{ hr}}$

The snail is slower than the sloth. [8 points]

4) $\frac{\$7.20}{1 \text{ doz}}$; the cost is $7.20 for every dozen bagels. [9 points]

5) $\frac{\frac{1}{30} \text{ gal}}{1 \text{ min}}$; water flows from the spring at a rate of $\frac{1}{30}$ gallon each minute. [8 points]

6) $\frac{\frac{11}{20} \text{ c}}{1 \text{ cookie}}$; it would take $\frac{11}{20}$ cup of flour to make 1 cookie. [9 points]

Shift 3: Rigor: Pursue conceptual understanding, procedural fluency, and application with equal intensity.

Lesson 3: Solve multistep problems involving unit rates. [36 points]

7) a. $\frac{57.4 \text{ ft}^2}{1 \text{ hr}}$; the painter paints at a rate of 57.4 square feet each hour. [6 points]

b. It will take him 2 hours to paint the bedroom. [6 points]

c. The painter would charge $50 to paint the bedroom. [6 points]

8) a. Train: $\frac{66\frac{3}{17} \text{ mi}}{1 \text{ hr}}$, or $66\frac{3}{17}$ mph; Bus: $\frac{55.01 \text{ mi}}{1 \text{ hr}}$, or 55.01 miles per hour [6 points]

b. Train: $\frac{\$33.45}{1 \text{ person}}$, or $33.45 per person; Bus: $\frac{\$33.50}{1 \text{ person}}$, or $33.50 for each person [6 points]

c. **Possible answer:** I think Alisa's class should take the train to get to the museum. [3 points]
Possible explanation: The train is a better choice for Alisa's class because it is a faster option, and it is slightly less expensive when compared with the unit rate for the class taking the bus. After I translated the information in the chart into math, I could easily compare the speed and price of taking the train or the bus. Because the unit rate for the train's speed is higher than the unit cost for the train's tickets, I know it is the better deal. [3 points]

Standard for Mathematical Practice
2: Reason abstractly and quantitatively.

Prep Points Analysis

Question Number	Core Objective	Team Scores (out of 20 points)					Class Results (check if 16 out of 20 points or better)
		1	2	3	4	5	
1	Find unit rates involving whole numbers and decimals.						
3	Find unit rates using data with fractions.						
5	Find unit rates using data with fractions.						
7a	Solve multistep problems involving unit rates.						
8b	Solve multistep problems involving unit rates.						

The mission of the Success for All Foundation is to develop and disseminate research-proven educational programs to ensure that all students, from all backgrounds, achieve at the highest academic levels.

These programs were originally developed at Johns Hopkins University.

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